Electrical Mathematics

Module One: Differentiation (John Bird Approach)

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Module One: Differentiation (Based on John Bird's Approach)

Introduction:

Differentiation is a fundamental tool in calculus, used to determine the rate of change of one variable with respect to another.

In electrical engineering, it is particularly useful for analyzing time-varying quantities such as current and voltage.

Learning Objectives:

- Understand the concept of the derivative as the rate of change.
- Learn basic rules of differentiation.
- Apply differentiation to polynomials and exponential functions.
- Understand its application in electrical circuit problems.

1. The Concept of a Derivative:

The derivative of a function y = f(x) with respect to x is defined as the limit of the average rate of change of the function over an interval as the interval approaches zero:

dy/dx = lim(h -> 0) [f(x + h) - f(x)] / h

2. Standard Derivatives (John Bird Table Summary):

- d/dx[x^n] = nx^(n-1)
- d/dx[sin x] = cos x
- d/dx[cos x] = -sin x
- d/dx[e^x] = e^x
- d/dx[ln x] = 1/x

3. Rules of Differentiation:

- Sum Rule: d/dx[u + v] = du/dx + dv/dx

- Product Rule: d/dx[uv] = u(dv/dx) + v(du/dx)

- Quotient Rule: d/dx[u/v] = (v(du/dx) - u(dv/dx)) / v^2

- Chain Rule: d/dx[f(g(x))] = f'(g(x)) * g'(x)

4. Worked Example (John Bird Style):

Example: Differentiate y = 3x^4 + 5x^2 - 6x + 2

Solution:

Using the power rule: dy/dx = (3×4)x^(41) + (5×2)x^(21) 6 + 0

=> dy/dx = 12x^3 + 10x 6

5. Application in Electrical Engineering:

- In an RC circuit, the voltage across the capacitor changes with time and is often expressed as V(t) = V0e^(-t/RC)

- The rate of change of voltage is given by dV/dt = -V0/RC * e^(-t/RC)

- This helps in determining time constants and analyzing transient response.

Practice Exercise:

1. Differentiate the following:

a) y = 4x^3 + 2x^2 - x + 5

b) V(t) = 10e^(-0.1t)

2. Find the rate of change of current if i(t) = 3t^2 + 2t

Conclusion:

Differentiation enables engineers to understand and predict changes in electrical quantities, essential for circuit design, control systems, and signal analysis.